Embedded Systems Design and Modeling

Chapter 13
Invariants and Temporal Logic

Correctness Definition

- Question: when is a design of a system “correct”?
- Answer: a design is correct when it meets its specification (requirements) in its operating environment
- Quotation: “A design without specification cannot be right or wrong, it can only be surprising!”
- To verify correctness, simply running a few tests is not enough!
- Many embedded systems are deployed in safety-critical applications (avionics, automotive, medical, …) and require rigorous verification

Examples From History

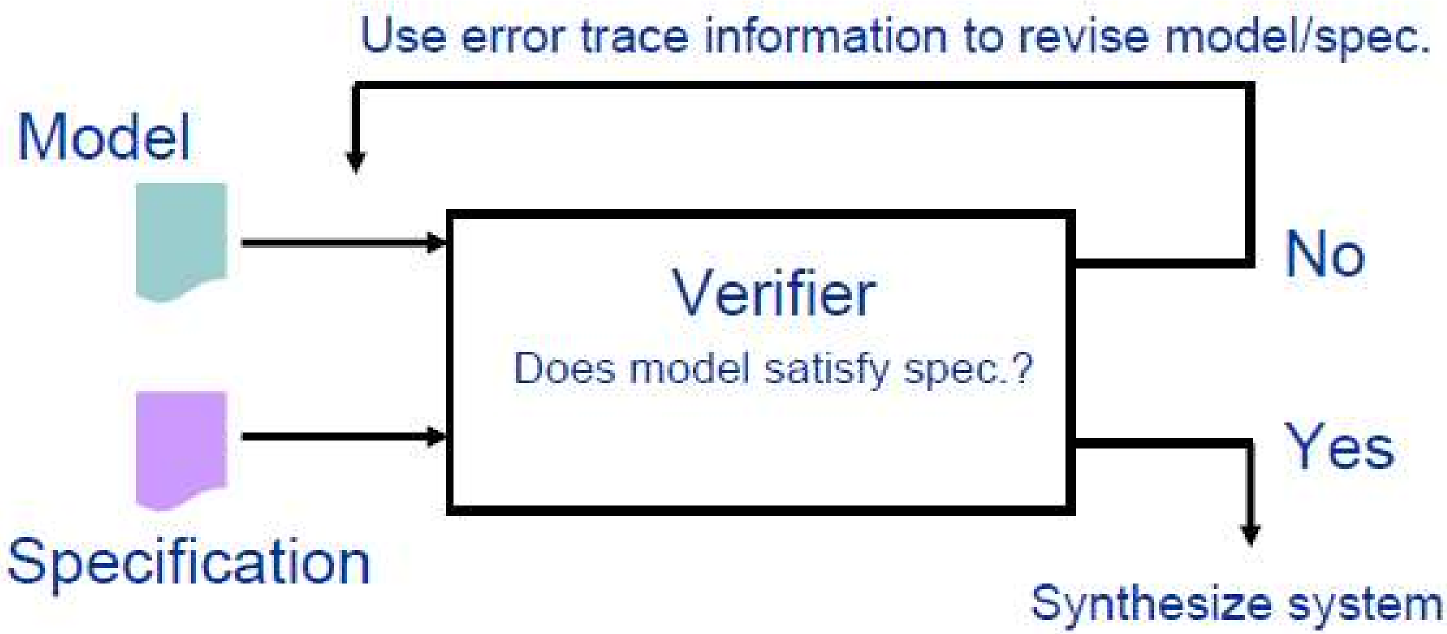

```
<msblast.exe> (the primary executable of the exploit)
I just want to say LOVE YOU SAN!!
billy gates why do you make this possible ? Stop
making money and fix your software!!
windowsupdate.com
start %s
tftp -i %s GET %s
%d.%d.%d.%d
```

Estimated worst-case worm cost:
> $50 billion

Basic Definitions

- Specification:
  - A precise mathematical statement of the design objective (desired properties of the system)
- Verification:
  - Does the designed system achieve its objectives in the operating environment?
- Controller Synthesis:
  - Given an incomplete design, a strategy to complete the system so that it achieves its objectives in the operating environment

Model-Based Design & Verification



- Requires a precise and unambiguous way to write models and specifications so that an algorithm can process it

Natural Language Deficiency

- Can natural languages satisfy this requirement?
- Generally no, due to their inherent ambiguities!
- Example: Specification of the SpaceWire Protocol (European Space Agency standard)

Note: The exact timing of this state is not specified clearly.

8.5.2.2 ErrorReset

a. The *ErrorReset* state shall be entered after a system reset, after link operation is terminated for any reason or if there is an error during link initialization.

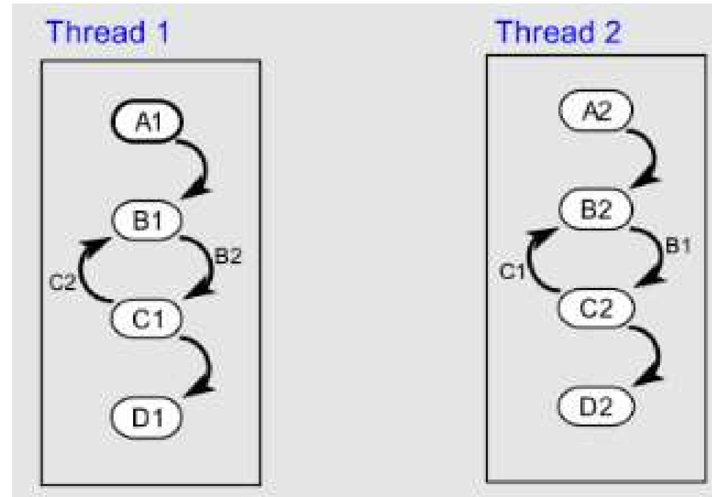
b. In the *ErrorReset* state the Transmitter and Receiver shall all be reset.

c. When the reset signal is de-asserted the *ErrorReset* state shall be left unconditionally after a delay of 6,4 µs (nominal) and the state machine shall move to the *ErrorWait* state.

d. Whenever the reset signal is asserted the state machine shall move immediately to the *ErrorReset* state and remain there until the reset signal is de-asserted.

Another Example

- Recall our previous example of mutual exclusion in a multithread system
- States and/or transitions represent atomic instructions
- Sample possible specifications described in a natural language:
  - The 2-threaded program should never be in state (C1,C2)
  - Thread 1 must eventually reach D1 and thread 2 must eventually reach D2



Motivational Observations

- Need a formal (mathematical) way (language) to specify the system.
- Various “logics” (mathematical languages) have been proposed to address this need.
- A mathematical specification only includes properties that the system must or must not have.
- It requires human judgment to decide whether that specification constitutes “correctness”.
- Getting the specification right is often as hard as getting the design right!

Temporal Logic

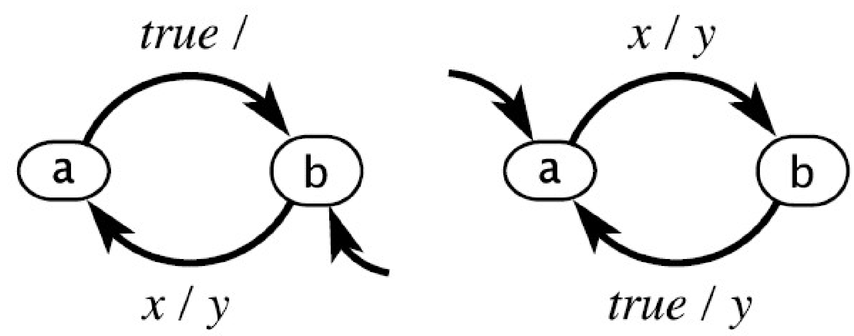
- A precise mathematical description to express properties of a system <u>over time</u>
  - E.g., Behavior of an FSM or Hybrid System
  - “Temporal” emphasizes the time aspect
- Many flavors of temporal logic:
  - Propositional temporal logic
  - Linear temporal logic
  - Real-time temporal logic, etc.
- ACM Turing Award was given for the idea of using temporal logic for specification

Propositional Temporal Logic

- Proposition: a statement about the inputs, outputs, or states of a system
- Can be seen as expressions with true or false values
- Atomic (smallest) proposition: fine-grained statements with as few as one single input or output or state
- A propositional logic formula (or simply a proposition) is a more elaborate combination of atomic propositions

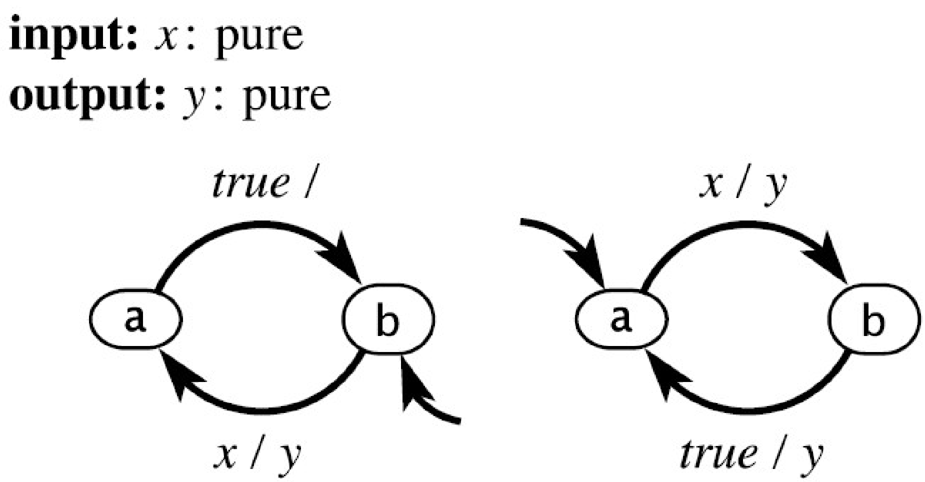
Atomic Propositions Example

input: x: pure
output: y: pure



true	Always true.
false	Always false.
x	True if input x is *present*.
$x = present$	True if input x is *present*.
$y = absent$	True if y is *absent*.
b	True if the FSM is in state b

Propositions Example



$x \wedge y$	True if x and y are both *present*.
$x \vee y$	True if either x or y is *present*.
$x = present \wedge y = absent$	True if x is *present* and y is *absent*.
$\neg y$	True if y is *absent*.
$\mathsf{a} \implies y$	True if whenever the FSM is in state a, the output y will be made present by the reaction

Execution Traces

An execution trace is a sequence of the form

$$q_0, q_1, q_2, q_3, \ldots,$$

where $q_j = (x_j, s_j, y_j)$ where s_j is the state at step j, x_j is the input valuation at step j, and y_j is the output valuation at step j. Can also write as

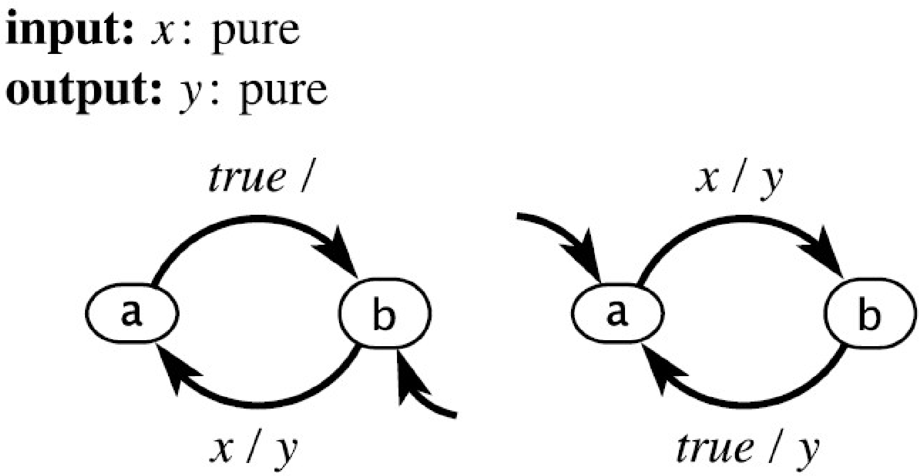
$$s_0 \xrightarrow{x_0/y_0} s_1 \xrightarrow{x_1/y_1} s_2 \xrightarrow{x_2/y_2} \cdots$$

Linear Temporal Logic

- LTL formula: applies to an entire trace instead of just one single element:
  - $q0, q1, q2, \ldots$
- If p is a proposition, then by definition, we say that LTL formula $\Phi = p$ holds for the trace $q0, q1, q2, \ldots$ if and only if p is true for $q0$.
- This may seem odd, but will provide temporal logic operators ways to reason about the entire trace.
- By convention, LTL formulas are denoted as Φ, $\Phi 1$, $\Phi 2$, etc. and propositions as p, $p1$, $p2$, etc.
- Given a state machine M and an LTL formula Φ, we say that Φ holds for M if Φ holds for all possible traces of M.
  - This typically requires considering all possible input combinations

LTL Example

- The LTL formula a holds for right hand side machine because all traces begin in state a.
- It does not hold for left hand side machine because there exists at least one trace that doesn't start in state a
- The LTL formula x => y holds for both machines because if x is present, then y will be present.
- The LTL formula y is false for both FSMs because there is a counterexample where x is absent in the first reaction.



LTL Formulas

formula	mnemonic	meaning
p	proposition	p holds in q_0
$\mathbf{G}\phi$	globally	ϕ holds for every suffix of the trace
$\mathbf{F}\phi$	finally, future, eventually	ϕ holds for some suffix of the trace
$\mathbf{X}\phi$	next state	ϕ holds for the trace $q_1, q_2, \cdots$
$\phi_1 \mathbf{U} \phi_2$	until	ϕ_1 holds for all suffixes of the trace until a suffix for which ϕ_2 holds.

G Operator

- Globally Φ:

The LTL formula $\mathbf{G}p$ **holds** for a trace

$$q_0, q_1, q_2, q_3, \ldots,$$

if and only if it holds for every suffix of the trace:

$$\begin{array}{l} q_0, q_1, q_2, q_3, \ldots \\ \quad q_1, q_2, q_3, \ldots \\ \quad\quad q_2, q_3, \ldots \\ \quad\quad\quad q_3, \ldots \end{array}$$

If p is a propositional logic formula, this means it holds for each q_i.

- Example: G(x =>y) is true for all traces of the right hand side machine, and hence holds for the machine.
- G(x ∧ y) does not hold for the machine, because it is false for any trace where x is absent in any reaction.

F Operator

- Finally Φ or eventually Φ:

The LTL formula $\mathbf{F}p$ holds for a trace

$$q_0, q_1, q_2, q_3, \cdots,$$

if and only if it holds for some suffix of the trace:

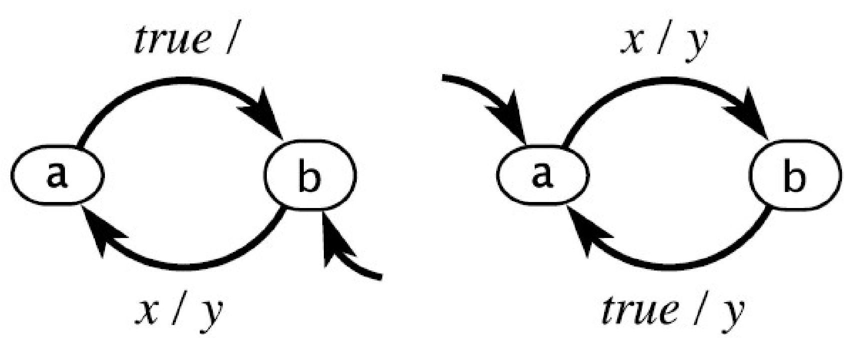
$$\begin{array}{r} q_0, q_1, q_2, q_3, \cdots \\ q_1, q_2, q_3, \cdots \\ q_2, q_3, \cdots \\ q_3, \cdots \end{array}$$

If p is a propositional logic formula, this means it holds for some q_i.

F Operator Examples

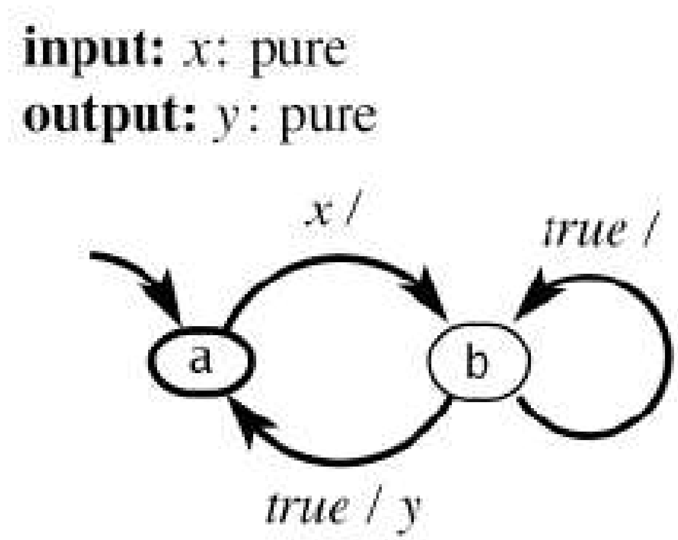
- G(x => Fb) holds for both machines:
  - If x is present in any reaction, then the machine will eventually be in state b.
  - True even in suffixes that start in state a.
- Parentheses (order) can be important in interpreting an LTL formula:
  - (Gx) => (Fb) is trivially true because Fb is true for all traces
- F¬Φ holds if and only if ¬GΦ:
  - (Φ is eventually false) = (Φ is not always true)

input: x: pure
output: y: pure



F Operator Examples (Continued)

- Does G(x => Fy) hold for this machine?
  - No because there is a counterexample in which y is not present even though x is present.



X Operator

- Next state Φ:

The LTL formula $\mathbf{X}p$ **holds** for a trace

$$q_0, q_1, q_2, q_3, \ldots,$$

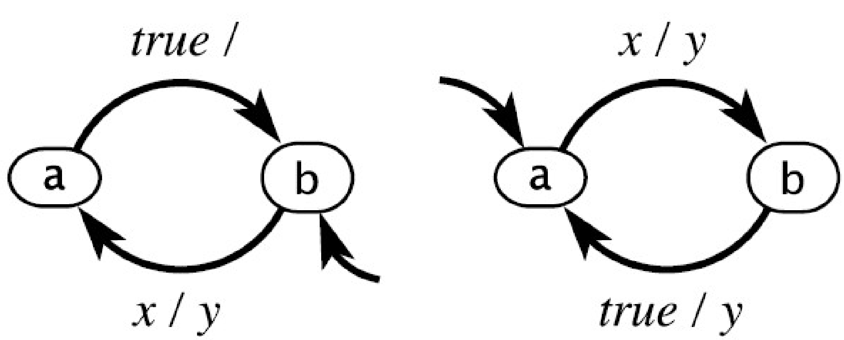
if and only if it holds for the suffix $q_1, q_2, q_3, \ldots$

$$q_0, q_1, q_2, q_3, \ldots$$
$$q_1, q_2, q_3, \ldots$$
$$q_2, q_3, \ldots$$
$$q_3, \ldots$$

X Operator Examples

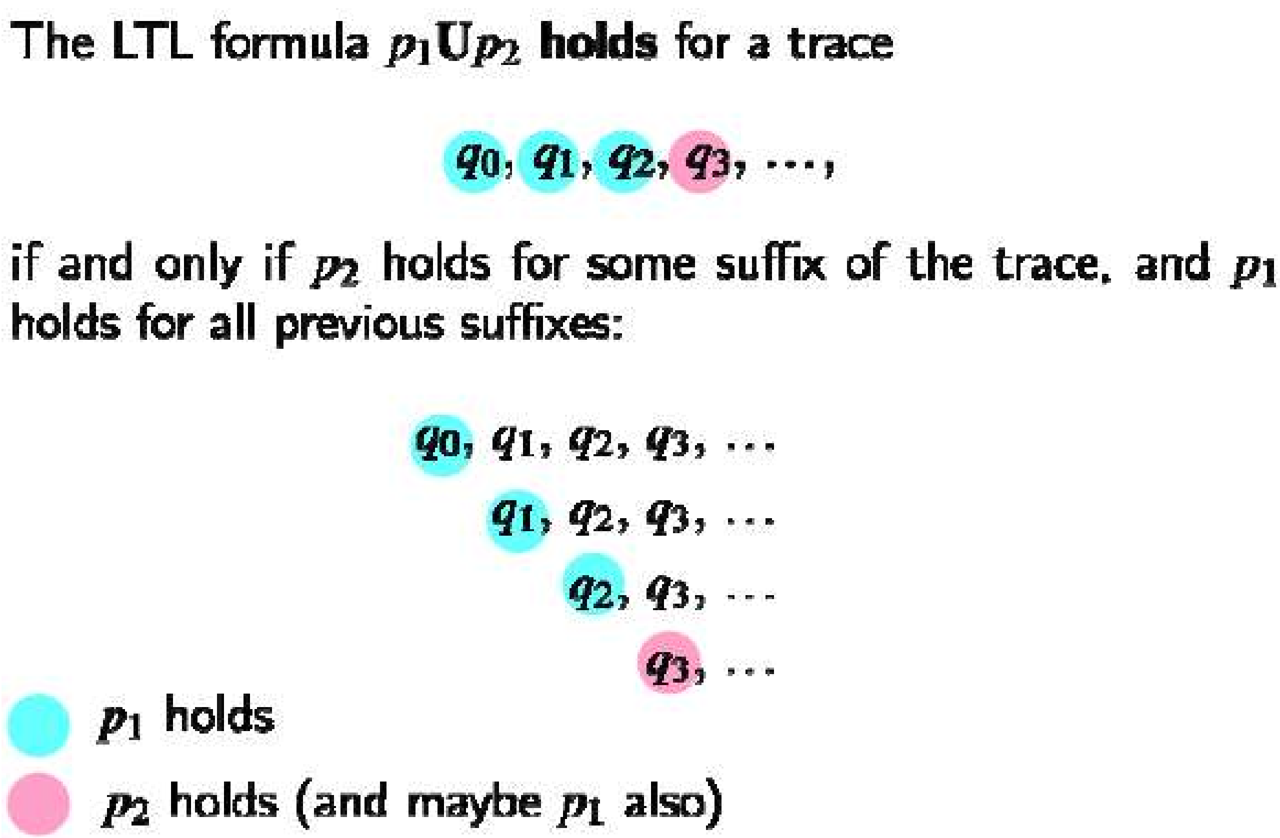
- x => Xa holds for the left side state machine:
  - If x is present <u>in the first reaction</u>, then the next state will be a.
- G(x => Xa) does not hold for the same state machine:
  - It does not hold for any suffix that begins in state a.
- G(b => Xa) holds for the right side state machine.

input: x: pure
output: y: pure



U Operator

- Until operator:



The LTL formula $p_1 \mathbf{U} p_2$ **holds** for a trace

$$q_0, q_1, q_2, q_3, \ldots,$$

if and only if p_2 holds for some suffix of the trace, and p_1 holds for all previous suffixes:

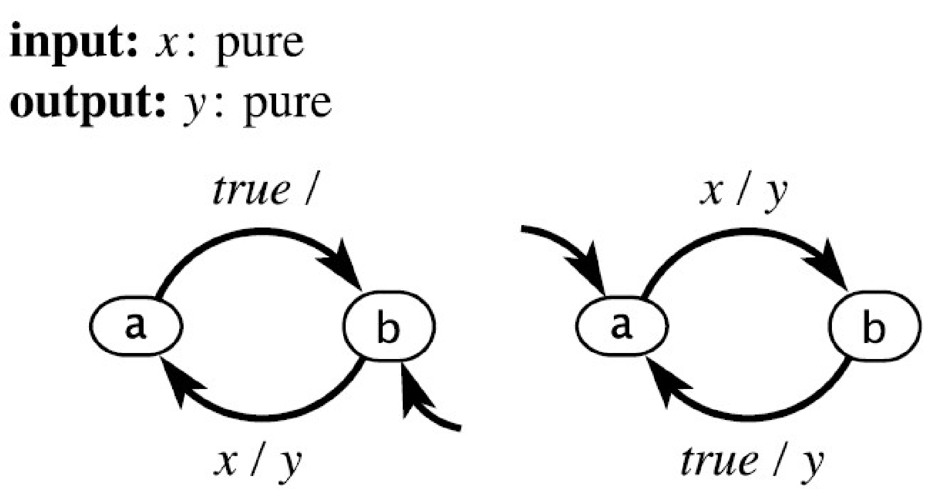
$$q_0, q_1, q_2, q_3, \ldots$$
$$q_1, q_2, q_3, \ldots$$
$$q_2, q_3, \ldots$$
$$q_3, \ldots$$

p_1 holds

p_2 holds (and maybe p_1 also)

U Operator Examples

- For the right side machine, aUx is true for any trace for which Fx holds.
- Since this does not include all traces, aUx does not hold for the state machine.



What do these mean?

- G F p:
  - p holds infinitely often
- F G p:
  - Eventually, p holds henceforth (steady state)
- G(p=> F q):
  - Every p is eventually followed by a q (request-response)
- F(p=> (XXq)):
  - If p occurs, then on some occurrence it is followed by a q two reactions later

Operator Relationships

- Can one express $G\Phi$ purely in terms of F, p, and Boolean operators?
  - Yes: $G\Phi = \neg F \neg \Phi$
- How about F in terms of U?
  - $F\Phi = \text{true}\ U\ \Phi$
- What about X in terms of G, F, or U?
  - Cannot be done!

Invariants

- An invariant is a property that holds for a system if it remains true at all times during operation of the system.
  - An invariant holds for a system if it is true in the initial state of the system, and it remains true as the system evolves, after every reaction, in every state.
- Example: In the model of a traffic light controller, there is no pedestrian crossing when the traffic light is green.
- This property must always remain true of this system, and hence is a system invariant.

Invariants (Continued)

- Invariant properties must include both software and hardware aspects of an embedded system
  - Software:
    - Correct programming style
    - Deadlock prevention in mutexes
    - Input data restrictions
    - etc.
  - Hardware:
    - Timing requirements
    - Data settlement issues
    - etc.

Homework Assignments

- Chapter 13:
  - Mandatory: 2, 4
  - Due date Tuesday 1404/2/30
  - The rest optional