Embedded Systems Design and Modeling

Chapter 2

Modeling

with an emphasis on

Continuous Dynamics

Part 2

System Properties: Causality

- Causal: output(s) depend only on current and past inputs (including that point)
- Need a mathematical representation:
  - A function "restriction in time" represents the current and past inputs
  - It is defined as: for times $t \leq \tau$

$$x_1|_{t\leq\tau} = x_2|_{t\leq\tau} \Rightarrow S(x_1)|_{t\leq\tau} = S(x_2)|_{t\leq\tau}$$

- Strictly causal: excluding that particular point in time (e.g., integrator actor)

$$x_1|_{t<\tau} = x_2|_{t<\tau} \Rightarrow S(x_1)|_{t\leq\tau} = S(x_2)|_{t\leq\tau}$$

System Properties: Memorylessness

- Memoryless: output(s) depend only on the current inputs (not the past inputs)
- Consider continuous time system S:

$$S : X \to Y, \text{ where } X = A^{\mathbb{R}} \text{ and } Y = B^{\mathbb{R}}$$

if there is a function f that for all x:

$$(S(x))(t) = f(x(t))$$

then the system is memoryless

- Integrator actor is NOT memoryless
- Adder actor is memoryless

System Properties: Linearity

- Linearity: a system S is linear if the superposition property holds:

$$\forall\, x_1, x_2 \in X \text{ and } \forall\, a, b \in \mathbb{R}, \quad S(ax_1 + bx_2) = aS(x_1) + bS(x_2)$$

  - Scale actor is always linear
  - Integrator actor is linear if initial value=0
- Linearity is a very important property in the study of systems because:
  - Combination of linear actors or systems (e.g., cascaded) will form another linear system

System Properties: Time Invariance

- Time invariant: when a system's behavior is independent of when the input is applied
- In other words, the system is repeatable
- Formal mathematical representation:
  - Define the delay actor:

$$\forall\, x \in X \text{ and } \forall\, t \in \mathbb{R}, \quad (D_\tau(x))(t) = x(t-\tau)$$

  - Time invariance:

$$\forall\, x \in X \text{ and } \forall\, \tau \in \mathbb{R}, \quad S(D_\tau(x)) = D_\tau(S(x))$$

- LTI: both linear and time invariant

System Properties: Stability

- Physical phenomena are bounded:
  - All inputs are bounded =>
  - All outputs have to be bounded in physical world too
- Unbounded behavior is perceived as instability
  - Example: natural frequency oscillations
- Stable system definition: for all bounded input signals output signals have to remain bounded too

System Properties: Stability (Cont'd)

- Formal mathematical definition:
  - Input, w(t), is bounded if there is a

    real number $A < \infty$ such that $|w(t)| \leq A$ for all $t \in \mathbb{R}$
  - Output, v(t), is bounded if there is a

    real number $B < \infty$ such that $|v(t)| \leq B$ for all $t \in \mathbb{R}$
  - A system is stable if for an arbitrary bound A, a bound B can be found for the output
- Integrator is NOT a stable system

Summary

- ODEs and actor models are examples of good MoCs
- Systems with continuous dynamics can be nicely described by them
- System properties are important parts of modeling embedded systems
- There are many challenges in the proper modeling, design, and analysis of well-known MoCs

Homework Assignments

- Chapter 2: 2, 3, 4, 5, 6, 7 for the next Tuesday 1403/11/30